

Regulating vehicle sharing systems through parking reservation policies: Analysis and performance bounds

Mor Kaspi, Tal Raviv, Michal Tzur, Hila Galili

Department of Industrial Engineering, Tel-Aviv University, Israel

August 2014

Abstract:

We study the regulation of one-way vehicle sharing systems through parking reservation policies. We measure the performance of these systems in terms of the total excess travel time of all users caused due to shortages of vehicles or parking spaces. We devise mathematical programming based bounds on the total excess travel time of vehicle sharing systems under any passive regulation and in particular under any parking space reservation policy. These bounds are compared to the performance of several partial parking reservation policies, a parking space overbooking policy, as well as the complete parking reservation (CPR) policy and the no-reservation (NR) policy that were introduced in a previous paper. A detailed user behavior model for each policy is presented and a discrete event simulation is used to evaluate the performance of the system under various settings. The analysis of two case studies of real-world systems demonstrates that: (1) a significant improvement of what theoretically can be achieved is obtained by the CPR policy. (2) The performance of the proposed partial reservation policies monotonically improve as more reservations are enforced. (3) Parking space overbooking is not likely to be beneficial. In conclusion, our results reinforce the effectiveness of the CPR policy and suggest that parking space reservations should be used in practice, even if only a small share of the users are required to place reservations.

Key words: vehicle sharing, bike-sharing, one-way car-sharing, reservations, performance bounds

1. Introduction

In recent years, vehicle sharing systems have become an integral part of the transportation services offered by many cities around the world. Such systems consist of a fleet of vehicles spread across the city, which users can rent for a short period of time. This type of service may be considered as an extension of the traditional public transport, which offers more flexibility and enables more multi-modal journeys. With this added flexibility, more citizens are able to shift from private vehicles to public transportation. In turn, this may result with a decrease in traffic congestion, a better utilization of land resources, especially in city centers (since fewer parking spaces are needed) and a reduction in air pollution and greenhouse gasses emissions.

In this study we focus on one-way, station based, vehicle sharing systems such as bike-sharing and some car-sharing systems. Such systems allow users to rent a vehicle in any of its stations scattered

around the city (given that there is an available vehicle in that station), use it for a short period of time and return it back to any station in which a parking space is available. In the case of bike-sharing systems, the "parking space" is in fact a docking pole. However, in the rest of the paper we use the term parking space to refer to one unit of storage of a vehicle of any kind. Some car-sharing systems are "free floating" (rather than station based). In such systems the vehicles can be rented and returned at any point in the city. These systems are not in the scope of this study. For a detailed description of the structure of vehicle sharing systems, the renting process, types of users and different business models, see surveys by: Shaheen and Cohen (2007, 2012), DeMaio (2009), Demaio and Meddin (2014), Shaheen et al. (2010) and Shaheen and Guzman (2011).

The operators of vehicle sharing systems face a difficult task of meeting the demand for vehicles and for available parking spaces. This difficulty arises mainly from the characteristics of the demand for journeys during the day. These demand processes are typically stochastic, a-symmetric and heterogeneous in time. The system is unable to satisfy the demand either when a user who wishes to rent a vehicle arrives at an empty station or when a user who wishes to return a vehicle arrives at a full station, i.e., a station with no vacant parking spaces. The latter scenario is typically perceived as more severe since a user who is unable to return a vehicle is "trapped" in the system because she cannot complete the renting transaction until she finds an available parking space. Contrary to that situation, a user who cannot rent a vehicle may decide to use an alternative mode of transportation.

Managers of public transport system should aim to improve the quality of service provided to its users, subject to the available resources. In this study, we measure the quality of service by the total excess time users spend in the system as a result of shortages in vehicles or parking spaces. The excess time of a user is the difference between the actual time she spends in the system (her exiting minus entering time) and her ideal time, i.e., the riding time between her origin and destination stations. Indeed, we believe that time is a major consideration of commuters in an urban public transit system. This is especially true in cities where regular commuters can buy a monthly or annual subscription for the transit system. We refer the reader to Kaspi et al. (2014) for a discussion on alternative performance measures, such as: the proportion of empty or full stations, the percentage of users who receive an ideal service and the percentage of users who do not use the system at all, i.e., those who abandon the system.

To reduce the occurrence of shortages in vehicles or parking spaces, the system operators may take either strategic or operational actions. Strategic actions include deploying more stations or enlarging existing stations, see for example, Lin and Yang (2011), Lin et al. (2011), George and Xia (2011), Correia and Antunes (2012) and Shu et al. (2013). Operational actions may include dynamically changing the fleet size and actively or passively regulating the system.

By active regulation we refer to redistributing vehicles between the system's stations using repositioning trucks (in the case of bikes-sharing systems) or by designated drivers (in the case of car-sharing). Raviv and Kolka (2013), Schuijbroek et al. (2013) and Vogel et al. (2014) devise methods for determining the desired daily initial inventories in the stations, that repositioning should aim to achieve. Kek et al. (2009), Nair and Miller-Hooks (2011), Benchimol et al. (2011), Angeloudis et al. (2012), Chemla et al. (2013a), Raviv et al. (2013), Erdoğan et al. (2013), Forma et al. (2014), and others, study static repositioning operations. Contardo et al. (2012), Kloimüller et al. (2014) and Pessach et al. (2014) study dynamic repositioning operations. However, repositioning of vehicles may be a costly procedure, especially in car sharing systems where each car is repositioned by a designated driver.

By passive regulation we refer to mechanisms used for redirecting demand in order to improve the performance of the VSS. Such mechanisms do not affect the true demand for journeys but may instead shift users to rent (return) vehicles at stations different from their true origin (destination) station. Fricker and Gast (2014) study a system regulation under which each user declares two optional destination stations and the system directs her to the less congested one. Several studies focus on pricing regulations as means of self-balancing vehicle sharing systems, see, for example, Chemla et al. (2013b), Pfrommer et al. (2013) and Waserhole et al. (2013). We note that the study of Waserhole et al. (2013) does not fall within our definition of passive regulations since they assume that the demand is elastic to the price.

In a previous paper, Kaspi et al. (2014) proposed implementing parking space reservations in one-way vehicle sharing systems in order to improve the performance of the system. In particular, they studied a complete parking reservation policy (CPR) in which all users are required, upon renting a vehicle, to reserve a parking space in their destination station. If there is a vacant parking space, it will be reserved for the user, and will not be available for other users from the moment the rent starts until the user returns the vehicle to the reserved parking space. If upon renting a vehicle there are no vacant parking spaces at the destination, the renting transaction will be denied. The user may then try to make a reservation at another station near her destination or may decide to use an alternative mode of transportation.

Under the CPR policy, a reserved parking space remains empty until the user returns her vehicle. In the meantime, other users cannot use this resource, i.e., it is blocked. The tradeoff in implementing such a policy is that while some users are guaranteed an ideal service (since they will certainly be able to return their vehicle at their desired destination) other users may receive poorer service due to the blocking of parking spaces. In Kaspi et al. (2014), the CPR policy was compared to the base policy entitled no-reservations (NR), using both a Markovian model with simplifying assumptions and an enhanced discrete event simulation model. Both policies are complete in the sense that all the users in the system are required to follow the same regulation. The results of the analysis demonstrated that the CPR policy outperforms the NR policy with respect to several service-oriented performance measures.

In this study we examine whether and to what extent a further reduction in the total excess time may be achieved by any other passive regulation, and especially by any other parking reservation policy. Towards that we use mathematical programming models to devise lower bounds on the total excess time that users spend in the system under any passive regulation and under any parking reservation policy. We consider the benefit of limiting the requirement to make a reservation to only some of the journeys. We refer to these policies as partial reservation policies, which combine the two extreme (complete) policies in different ways. We evaluate the performance of all policies and compare them to the lower bounds.

The contribution of this paper is as follows. First, using mathematical programming models, we provide for the first time lower bounds on the performance of a VSS, measured by the total excess time, under any passive regulation and under any parking reservation policy. Second, we introduce the concept of partial reservation policies. We examine three different partial policies, each is based on a simple sound principle, which is easy to control by the system's managers and communicate to the users. We define the user behavior under these policies and examine their performance using discrete event simulation of real world systems. Third, we examine the potential benefit of parking space overbooking.

Our results indicate that, surprisingly, the CPR policy achieves a significant part of the improvement that can theoretically be attained by any passive regulation, compared to applying no regulation (the NR policy). In addition, we show that under three reasonable partial reservation policies the performance of the system is inferior to its performance under CPR. In fact, we were unable to come up with a partial reservation policy that outperforms the CPR. This reinforces the advantage of implementing the CPR policy. Nevertheless, our numerical study shows that the examined partial policies can significantly improve the performance of the system compared to the NR policy. Therefore, if, due to any considerations or constraints, the system managers are reluctant to implement CPR, then most of its advantages can be obtained by implementing a partial reservation policy. Finally, parking space overbooking is ruled-out as a direction for improving the results obtained by the CPR policy. We show that on one hand it cannot yield a significant improvement in terms of the expected performance and on the other hand it introduces some undesirable uncertainty to the system.

The paper is organized as follows: in Section 2 a generic description of the VSS is presented and mathematical models are formulated to bound its performance under passive regulations and in particular under parking reservation policies. In Section 3 a behavior model of users in vehicle sharing systems is presented and the proposed partial parking reservation policies are described. A utopian overbooking policy is presented at the end of this section. A description of two real world vehicle sharing systems and numerical results for their performance are presented and discussed in Section 4. Concluding remarks are provided in Section 5.

2. Lower bounds on the total excess time in a VSS

As mentioned in the introduction, the operational actions that a VSS operator can take in order to provide a high quality service can be of two forms, namely, active regulations and passive regulations. In this study we focus on passive regulations, i.e., mechanisms used for redirecting demand.

VSS are decentralized systems, that is, each user makes decisions regarding her planned itinerary so as to minimize her own expected excess time. Such decisions depend on the availability of vehicles or parking spaces at the system's stations at the renting time as well as on the user's expectations regarding future availability. In addition, the user's decisions are subject to the passive regulation prescribed by the system. Under passive regulations, the system may influence the users' decisions by limiting their choices or by incentivizing them to prefer certain itineraries. However, the system does not assign itineraries to the users. For example, under the CPR policy, if a user cannot make a parking reservation at a certain station, she is not allowed to travel with a shared vehicle to that station, but the choice of her actual alternative itinerary is hers. From the operator's point of view the question is: how to design a passive regulation so that the outcome of all users' decisions minimizes the expected total excess time?

A passive regulation can be formally defined as a mapping of the state of the system and the demand for journeys to a set of itineraries allowed for each journey. The set of possible passive regulations is enormous. However, a major share of these regulations may be hard to implement or hard to communicate to the users. In this study we introduce and analyze regulations in the form of parking reservation policies, which are based on simple principles and are easy to communicate to the users. In order to assess the potential improvement that may be achieved by passive regulations in terms of the expected total excess time, we formulate mathematical programs that provide lower bounds. First, we devise a lower bound on the expected total excess time under any passive regulation. Second, since this study focuses on parking reservation policies, we devise a tighter bound specifically for any parking reservation policy.

The rest of this section is organized as follows: Section 2.1 includes a description of a VSS, and assumptions concerning the demand. Section 2.2 contains a MILP formulation whose optimal value is a lower bound on the excess time that may be achieved by any passive regulation. Section 2.3 modifies the MILP formulation to account for passive regulations that involve only parking reservations, thus obtaining a tighter bound.

2.1. Description of the VSS

In this section we discuss the information needed in order to model a vehicle sharing system. Such information is used both in the mathematical models that are presented in this section, and in the user behavior model presented in Section 3. The information needed in order to describe the system is as follows:

- The number of stations in the system
- The number of parking spaces in each station (referred to as the capacity of the station)
- The initial inventory level (number of vehicles) at each station
- The expected travel time between any pair of stations using the shared vehicles
- The expected travel time between any pair of stations, using an alternative mode of transportation.

Note that information about locations of the stations is not needed. In order to describe the relations between the stations it is enough to specify the traveling times between each pair of stations. The distance between the stations, the topography of the city, congested roads and other considerations are all taken into account in the traveling times. In some situations, due to shortages in vehicles or parking spaces, users may roam to nearby stations (using an alternative mode of transportation) or may decide to abandon the system altogether and make their entire journey using an alternative mode of transportation. Therefore, the traveling times between any pair of stations using an alternative mode of transportation are also needed. For example, in bike sharing systems a reasonable assumption is that the alternative mode is walking.

Each demand for a journey is defined by the desired origin and destination stations and by the desired starting time. An underlying assumption of all the models introduced in this paper is that all the journeys in the system start and end at stations of the VSS. An alternative mode of transportation is always available to the users while the shared vehicle (resp., parking space) may or may not be available at the origin (resp., destination).

2.2. A lower bound on the total excess time under any passive system regulation

Our goal in this section is to establish a lower bound on the total excess time that results from the users' decisions under any passive regulation. Given the system's characteristics and a demand realization for journeys during a predetermined planning horizon (typically a day), we formulate an optimization problem that centrally selects the itineraries of the VSS's users so as to minimize the total excess time.

The solution value of this optimization problem is a lower bound on the total excess time that may be achieved by any passive regulation, due to the following two assumptions on which the optimization problem is based:

1. All demands for journeys are known in advance. This is clearly information that is not available in practice.
2. A central planner determines the itinerary of each user, in a way that benefits the entire system. In practice, each user determines her own itinerary, according to her individual objective and given the information she has. The justification of this assumption is that any solution selected by the central planner may be selected by the users under some passive regulation.

Due to the system's limited resources, this bound is typically strictly positive, thus it is better than the trivial bound of zero excess time (no shortages of any type).

In practice the demand for journeys is a stochastic process. Therefore, the average solution value of the optimization problem for numerous demand realizations, drawn from a given stochastic process, provides an estimator for a lower bound on the expected total excess time under any passive regulation.

A demand realization is described by a set of journeys where each journey is characterized by an “origin-destination-time” tuple. Each journey can be materialized by one of several possible itineraries. We assume that a possible itinerary is in one of the following forms:

1. Use a **shared vehicle** all the way from the origin to the desired destination.
2. Use an **alternative mode** of transportation to get to a station with an available vehicle and then use a **shared vehicle** from this station to the desired destination.
3. Use an **alternative mode** of transportation to get to a station with available vehicle and then use a **shared vehicle** from this station to another station with an available parking space. Then, from this station use an **alternative mode** to get to the desired destination.
4. Use a **shared vehicle** from the origin station to another station with an available parking space and then use an **alternative mode** to get to the desired destination.
5. Use an **alternative mode** all the way from the origin to the desired destination.

We refer to the station where the vehicle is actually rented (resp., returned) as the renting (resp., returning) station. Upon attempting to return a shared vehicle, the user may be required to wait at the returning station until a parking space becomes available, and then proceed with her itinerary (leave the system or continue with an alternative mode of transport).

Each possible itinerary can be defined by its renting station, returning station and renting time. The returning time is determined by the renting time and the riding time between the two stations. Waiting times at the returning stations are not considered as part of the itinerary times, they are calculated separately. In addition, a journey can be materialized by an itinerary that includes only an alternative mode of transportation. Clearly, such an itinerary is not associated with renting and returning stations.

We further define a set of possible events, where each event is a time-station tuple that refers to a renting or returning time and a location of one or more of the possible itineraries. The events of each station can be ordered by their times. At the time of each event the state of the corresponding station is defined by the number of vehicles parking at the station and the number of users that are (possibly) waiting to return their vehicles at the station. Next, the notation required to formulate the optimization model is presented.

Indices:

s	Station
t	Time
j	Journey
i	Itinerary

Parameters:

S	Set of stations
J	Set of journeys
C_s	Capacity of station s
I_s^0	Initial vehicle inventory in station s
E	Set of possible events $[(s, t)$ tuples]
\mathbb{I}_j	Set of possible itineraries of journey j , we also use $\mathbb{I} \equiv \bigcup_{j \in J} \mathbb{I}_j$
X_i	The excess time of itinerary i
$W_{(s,t)}$	The time difference between event (s, t) and the next event at station s
$B(s, t)$	Set of itineraries in which a vehicle is rented at station s at time t
$F(s, t)$	Set of itineraries in which a vehicle is returned at station s at time t
$\mathcal{P}(s, t)$	The event that precedes event (s, t) at station s

In addition, we define two artificial events $(s, 0)$ and (s, H) for each station s , that denote the beginning and the end of the planning horizon, respectively.

Decision variables:

r_i	1 if itinerary i is selected, 0 otherwise
$p_{(s,t)}$	Number of vehicles parking at station s right after event (s, t)
$w_{(s,t)}$	Number of users waiting to return a vehicle at station s right after event (s, t)

The problem can now be formulated as a mixed integer linear programming model. We refer to this model as the Passive Regulation Lower Bound (PR-LB).

$$\text{minimize } \sum_{i \in \mathbb{I}} X_i \cdot r_i + \sum_{(s,t) \in E} W_{(s,t)} \cdot w_{(s,t)} \quad (1)$$

Subject to

$$\sum_{i \in \mathbb{I}_j} r_i = 1 \quad \forall j \in J \quad (2)$$

$$p_{\mathcal{P}(s,t)} + w_{\mathcal{P}(s,t)} + \sum_{i \in F(s,t)} r_i = p_{(s,t)} + w_{(s,t)} + \sum_{i \in B(s,t)} r_i \quad \forall (s,t) \in E \quad (3)$$

$$p_{(s,0)} = I_s^0 \quad \forall s \in S \quad (4)$$

$$p_{(s,t)} \leq C_s \quad \forall (s,t) \in E \quad (5)$$

$$w_{(s,0)} = 0 \quad \forall s \in S \quad (6)$$

$$w_{(s,H)} = 0 \quad \forall s \in S \quad (7)$$

$$r_i \in \{0,1\} \quad \forall i \in \mathbb{I} \quad (8)$$

$$p_{(s,t)} \geq 0 \quad \forall (s,t) \in E \quad (9)$$

$$w_{(s,t)} \geq 0 \quad \forall (s,t) \in E \quad (10)$$

The objective function (1), sums over the excess time of the selected itineraries and the waiting times of all the users who wait to return their vehicles at their returning station. Constraints (2) assure that for each journey exactly one itinerary is selected. Constraints (3) are vehicle inventory balance equations. Constraints (4) set the initial vehicle inventory in each station. Constraints (5) limit the number of parked vehicles in a station to the capacity of the station. Constraints (6) and (7) state that no user is waiting to return a vehicle at the beginning or at the end of the planning horizon. Constraints (8) stipulate that the itinerary decision variables are binary. Constraints (9) and (10) are non-negativity constraints on the number of parked vehicles and waiting users after each event.

In this model the central planner may assign a user with any of its given potential itineraries. In some cases the users may be referred to relatively distant rent or return stations, merely in order to balance the vehicle inventories, for the system's benefit, and not necessarily because the system cannot satisfy their demand by better itineraries. In the next section we extend the model to limit such occurrences.

Theoretically, a user may begin her ride and return the vehicle at any station. Therefore, the number of potential itineraries of a journey is the square of the number of stations. However, most of these potential itineraries would take longer than simply using the alternative mode of transportation for the entire journey (i.e., abandoning the system). Under many regulations it is safe to assume that users will not accept such itineraries. In the numerical experiment reported in Section 4, we let the central planner consider only potential itineraries that are not longer than using the alternative mode of transportation for

the entire journey. Moreover, in order to reduce the computational effort required for solving the PR-LB model (1)-(10), we relaxed the integrality constraints (8) and replaced them with non-negativity constraints. This clearly preserves the result as a lower bound. In our numerical experiment we observed that the effect of this relaxation on the obtained lower bound is negligible.

Though this study focuses on parking reservation policies, the above model serves as a lower bound on the excess time under any passive regulation. In particular, since the input for this model consists of all the demand for journeys it can also serve as a bound for vehicle or trip reservation policies.

2.3. A lower bound on the total excess time under any parking reservation policy

In this section we focus on a subset of all possible passive regulations, namely, parking reservation policies. An important feature of such policies is a parking space guarantee provided to some users for part or the entire duration of their journey. This potential guarantee is accompanied by possible restrictions the system may impose on some journeys. The system must allow a user to rent a vehicle if one is available at the desired renting station at the requested time. In addition, the system must allow a user to travel to a returning station if a vacant parking space is available at that station at the renting time. However, the system may prevent users from traveling to stations in which there is no vacant parking space at the renting time. Whether this would be allowed or not depends on the setting of the parking reservation policy. For example, under the CPR policy, this is always forbidden.

The above restrictions limit the set of potential itineraries that may be materialized under any parking reservation policy. The PR-LB model (1)-(10) is modified such that the central planner may assign to each journey an itinerary from this restricted set, which is acceptable by the user. Namely, we exclude itineraries that a user who minimizes her own excess time would not select under any parking reservation policy. The following itineraries may be selected by a user: (a) the shortest itinerary for which a vehicle is available at the renting station at the renting time and a parking space is available at the returning station at the renting time; (b) the itinerary under which an alternative mode of transportation is used for the entire journey, i.e., abandoning the system; (c) any itinerary whose length is not longer than (a) and (b), for which a vehicle is available at the renting station at the renting time. It is clear from the definition of (a), that for itineraries included in (c), no parking space is available at the returning station at the renting time. Such an itinerary may be beneficial if a parking space becomes available before or shortly after the user arrives at the returning station. We note that since itineraries that belong to (c) are allowed, the model indirectly considers all possible parking reservation policies. For this reason the optimal solution of the model presented below is a lower bound for any parking reservation policy.

The restricted set of potential itineraries per journey cannot be pre-defined as an input to the model. This is because the selection of the itineraries that are included in the set depends on the system's state at

the decision time and on all the decisions that were made for journeys that begin prior to that journey. Instead, we modify the PR-LB model (1)-(10) by adding sets of decision variables and constraints so as to properly exclude potential itineraries that a user will definitely not choose. We refer to this extended model as the Parking Reservation Policy Lower Bound model (abbreviated PRP-LB).

We use the same notation as in the PR-LB model (1)-(10) and add the following parameters and decision variables:

Parameters:

- $O(i)$ An (s, t) tuple that represent the renting station and renting time of itinerary i
- $D(i)$ An (s, t) tuple that represent the returning station and returning time of itinerary i
- $J(i)$ The journey that can be materialized by itinerary i
- $T(s, t)$ Time of node (s, t)
- $S(s, t)$ Station of node (s, t)
- \mathcal{R}_i A set of itineraries for which a parking space may be reserved at the returning station of itinerary i at the renting time of itinerary i . That is, an itinerary k is in the set if:
 - It is of a different journey, $J(k) \neq J(i)$
 - It has the same returning station as itinerary i , $S(D(k)) = S(D(i))$.
 - The renting time of itinerary k is earlier than the renting time of itinerary i , $T(O(k)) < T(O(i))$
 - The returning time of itinerary k is later than the renting time of itinerary i , $T(D(k)) > T(O(i))$
- M A very large number (for example, twice the capacity of the largest station)

Auxiliary decision variables:

- $e_{s,t}$ 0 if a vehicle is available at station s at time t , otherwise it can either obtain 0 or 1.
- f_i 0 if at renting time $T(O(i))$ there are some vacant parking spaces at station $(D(i))$, otherwise it can obtain either 0 or 1.

The PRP-LB model can be stated now as (1)-(10) with the following additional constraints:

$$e_{O(i)} + f_i \geq r_k \quad \forall i, k \in \mathbb{I}_j : X_i < X_k \quad \forall j \in J \quad (11)$$

$$M \cdot (1 - e_{(s,t)}) \geq p_{(s,t)} + w_{(s,t)} \quad \forall (s, t) \in E \quad (12)$$

$$C_{S(D(i))} \cdot f_i \leq p_{D(i)} + w_{D(i)} + \sum_{k \in \mathcal{R}_i} r_k \quad \forall i \in \mathbb{I} \quad (13)$$

$$e_{(s,t)} \in \{0,1\} \quad \forall (s, t) \in E \quad (14)$$

$$f_i \in \{0,1\} \quad \forall i \in \mathbb{I} \quad (15)$$

Constraints (11) stipulate that each journey is materialized by the shortest possible itinerary, i.e., the one with the shortest excess time. For any itinerary k , if an itinerary i of the same journey with shorter excess time exists, a vehicle is available at the renting station of i ($e_{O(i)}=0$) and a parking space is

available at its returning station at its returning time ($f_i = 0$) then itinerary k cannot be selected ($r_k = 0$). By constraints (12), a station can be considered "empty" in a given time, only if there are no vehicles parked or waiting in it at that time. Constraints (13) assure that the f_i variables are set to zero if there are vacant parking spaces at the returning station of itinerary i at the renting time, i.e., parking spaces that are not occupied and are not reserved. Constraints (14) and (15) stipulate that the variables $e_{(s,t)}$ and f_i are binary.

The value of the solution of the PRP-LB model (1)-(15) provides a tighter bound on the total excess time as compared to the PR-LB model (1)-(10). This is because it is based on a super-set of its constraints and since parking reservation policies are a subset of any passive regulation. As in the case of PR-LB, this model was solved while relaxing the binary variables r_i . The binary variables $e_{(s,t)}$ and f_i were not relaxed since without these constraints the resulting relaxation is very weak. This is due to the fact that in constraints (12) and (13) these variables are multiplied by large constants. Indeed, this model is more difficult to solve, see Section 4.

In the PRP-LB model, unlike the PR-LB, if vehicles are available at the station at the arrival time of a renter, the system must offer one to the user. Therefore, this model cannot provide a lower bound for the performance of vehicle or trip reservation policies.

3. Parking reservation policies

The lower bounds developed in the previous section may be used to evaluate the effectiveness of any regulation or parking reservation policy. In this section, we introduce several parking reservation policies. The performance of a VSS under these policies or under any other regulation can be evaluated only with respect to the response of the users to the rules prescribed by the regulation. We base our analysis, with respect to users' response, on an axiomatic approach and model the users as rational independent agents whose goal is to minimize their own excess time. However, achieving this goal may be too complex for many users due to the stochastic nature of the VSS. Therefore, we postulate a user behavior model that heuristically approximates this minimization problem and, in fact, provides an optimal solution in most of the cases.

In Section 3.1 we present this user behavior model. The model describes the decisions taken by the users at different decision points. These decisions are affected by the state of the system and the settings of the regulation. In Section 3.2 we present three partial reservation policies, discuss the motivation for using them and explain how they are reflected in the user behavior model. In Section 3.3 we present a utopian parking overbooking policy, whose performance is used to gauge the potential benefit of parking overbooking policies.

3.1. User behavior model

The movement of users in the system depends both on its regulation and on the state of the system (availability of vehicles and parking spaces). A user who enters the system acts as follows: If there are no available vehicles at her origin station, she may either decide to go to a nearby station, using an alternative mode of transportation, in search for an available vehicle, or she may decide to abandon the system. An abandoning user is assumed to travel to her destination using an alternative mode of transportation. Note that in modern VSS, the user can make this decision based on real time information about the availability of vehicles in the stations of the system. Once a user finds an available vehicle, there are two options: (1) A parking reservation is not required and (2) a parking reservation is required. In option (1), the user rents a vehicle and travels to her destination. When the user reaches her destination (with a vehicle), if she finds an available parking space she returns the vehicle and exits the system. If there is no available parking space at the destination station, the user may decide to wait at the station until a parking space becomes available (i.e. she enters a waiting queue). Alternatively, the user may decide to roam to a nearby station in search for an available parking space. Again, this decision is based on real time information on the availability of parking spaces in the stations. In option (2), the user attempts to make a parking reservation in her destination station. If the reservation is approved, the user makes a rent-and-reserve transaction and travels to her destination station. If the parking reservation is guaranteed, the user can immediately exit the system upon reaching her destination. If the reservation is not guaranteed, the user travels to the returning station and proceeds as in option (1). If the parking reservation is not approved, the user can either try to make a reservation at another station near her destination or she may decide to abandon the system. Finally, if, for some of the above reasons, the vehicle is returned to a station different than the user's destination, the user uses an alternative mode of transportation to reach her destination station and then exits the system.

This behavior model is described in Figure 1. At decision points, we assume that users have full knowledge of the system state, including inventory levels at each station and the arrival rate of renters to each station (for instance using smartphone applications). Users are assumed to be strategic so that at decision points they will choose the alternative that minimizes their expected remaining traveling time.

station and using an alternative mode of transportation from there to the destination is the shortest among all stations with available parking spaces. The user would choose to wait in the station until a parking space becomes available, if the expected time until this happens is shorter than the above alternative.

III. A renter who cannot make a parking reservation at the destination station would consider making reservation at a nearby station such that the total time spent traveling with the shared vehicle to the chosen returning station and using an alternative mode of transportation from there to the destination is the shortest among all stations for which it is possible to make parking reservation. The user would choose using an alternative mode of transportation for the entire journey if it shorter than the above alternative.

In the user behavior model, there are three junctions that represent the settings of the policy:

- Parking reservation required?
- Reservation approved?
- Vacant parking space guaranteed?

To highlight these junctions, we plot them in Figure 1 as trapezoids. The NR and CPR policies, are *complete* in the sense that under each of these policies the answer to each of the above three questions is identical for all the users of the system. For example, under the CPR policy, all users are required to make a parking reservation, the reservation is approved if at the renting time there is an available parking space at the returning station and a vacant parking space is guaranteed to all users who are able to make a parking reservation.

3.2. Partial parking reservations policies

In this section we present three types of partial parking reservation policies. Each type is based on a simple, yet reasonable principle. The common motivation for these policies is to enforce parking reservations only when they are likely to make a positive effect on the performance of the system. In the descriptions below, a *trip* is defined as a direct ride between a pair of origin-destination stations.

3.2.1. Trip based partial reservation policy

Under this policy, parking reservations are required only for trips with expected traveling times shorter than a given threshold. At rent, a user will declare her destination and if the expected traveling time is shorter than the given threshold she will be required to reserve a parking space in her destination. As in the CPR policy, if at the renting time there is no vacant parking space at the destination, the transaction is denied and the user may try to make a reservation at a different station. A user with expected riding time longer than the threshold time, will not be required to make a parking space reservation. If such a user

finds an available vehicle at her origin, she will be able to rent it and ride to her destination, as in the NR policy. The rationale behind this policy can be stated as follow: since a parking space is a valuable resource in a VSS and a reservation practically blocks it for the duration of trip, the parking space should be reserved only for short trips.

Note that if the threshold time is set to zero, this policy coincides with the NR policy. On the other hand, if the threshold is set to a large enough value this policy coincides with the CPR policy. Different partial polices of this type can be obtained by setting the value of the threshold parameter between the two extremes.

3.2.2. Station based partial reservation policy

Under this policy a parking reservation is required if the difference between the expected returning and renting rates at a certain time interval is higher than a pre-specified *difference threshold*. If the calculated difference is lower than the difference threshold the user will behave as under the NR policy. Expected renting and returning rates can be estimated using past transactions.

The rationale behind this policy can be stated as follows: the probability of parking space shortages in a station grows as the imbalance (difference) between the demand rates for parking spaces and for vehicles grows. Such imbalances may be consistent, for example, in bike sharing stations at relatively low altitude locations, bicycles are more likely to be returned than rented. Or, the imbalance may change during the day, for example, in stations located at working areas where returning in the morning is much more prevalent than renting. When the demand rate for parking spaces (returning) is higher than the demand rate for vehicles (renting), there is a greater chance that users will find the station full. By enforcing parking reservations at such stations, the system prevents users from traveling to stations with no available parking spaces. This will shift some of the users to nearby, less congested stations. Such a shift would have probably happened anyway, since users who find a full station typically roam to a nearby station in order to return their vehicle. With parking reservations, the change in the returning rate is determined in advance, which is likely to reduce the users' excess time. On the other hand, it seems less effective to enforce parking reservations in stations that are likely to be empty anyway.

Note that the higher the difference threshold is, the fewer the cases in which reservations are required. For extremely high threshold values the policy coincides with the NR policy, while for extremely low (negative) values it coincides with the CPR policy.

3.2.3. Time limited partial reservation policy

Under this policy all users are required to make a parking reservation, as in the CPR policy, but this reservation is valid for a limited time. After the reservation expires, the reserved parking space becomes

available to other users, and a vacant parking space is no longer guaranteed to the user. If the reservation expires and no parking space is available by the time the user arrives at the destination station, she will have to either wait by the station or roam to a nearby station (as in the NR policy).

The rationale behind this policy can be stated as follows: by making a reservation, a user with a long traveling time that reaches her destination only after her reservation expires, still affects the system because as long as her reservation is valid, she may block other users from making a reservation. That is, the reservation may divert subsequent demand, which may increase the probability of the user to find a vacant parking space, even if her reservation expired.

Note that if the time limit is set to a large enough value, this policy coincides with the CPR policy. However, if the time limit is set to zero, the resulting policy is still different from the NR policy. This is because users still must make a reservation, and they cannot begin traveling to a station that is full at the renting time. In Section 4 we compare the performance of this specific setting (in which the time limit is zero) to the performance of the NR policy and discuss the differences and their implications.

In Table 1 we summarize the answers to each of the three questions that appear in the user behavior model, which characterize the settings of the parking reservation policies described above. For completeness, this table also includes the information regarding the utopian parking space overbooking policy, described in the next section.

Table 1: Settings of the various parking reservation policies

Parking reservation policy	Parking reservation requirement	Conditions to approve a parking reservation	Vacant parking space guaranteed
NR	For none of the users	--	--
CPR	For all users	A vacant parking space at the destination at the renting time	Yes
Partial: trip based	For users with trip time shorter than a given threshold	A vacant parking space at the destination at the renting time	Yes
Partial: station based	For users with a destination station in which the difference between the returning rate and the renting rate is higher than a given threshold	A vacant parking space at the destination at the renting time	Yes
Partial: time limited	For all users	A vacant parking space at the destination at the renting time	Only for users with trip time shorter than the time limit
Utopian overbooking	For all users	The system anticipates that there will be a vacant parking space at the destination at the returning time	Yes, in this hypothetical utopian setting the user is allowed to return a vehicle even when no parking space is available

3.3. Utopian parking space overbooking policy

In many service systems in which reservations are carried out, it is a common practice to allow overbooking. It means accepting reservations for resources that, based on previous reservations, will not be available at the required time. Overbooking may be an effective policy in the presence of stochasticity in the arrival or service process, for example, no-shows of customers who made reservations. In a vehicle sharing system that practices parking reservations, no-shows are not an issue because the reservations are made at the renting time and the users must return the vehicle at the stated destination. Nevertheless, it might be beneficial to allow in some cases for users to travel to a station even if it has no available parking spaces, or in other words, to allow overbooking. This is because a parking space may become available by the time the user reaches her destination station. A good overbooking policy is based on reliable forecast that is capable of predicting such occurrences.

In order to evaluate the potential benefit of overbooking policies, we envision a system that has full information regarding the demand for vehicles at the station for which it considers allowing overbooking. The overbooking decisions will be based on this information, thus it is referred as a utopian overbooking policy. Note however, that this policy optimizes the service provided to each user individually rather than taking the system point of view as in the lower bounds presented in Section 2.

Under this policy, upon renting the user is required to declare her destination, and then the system decides whether a reservation can be made or not. The system's decision is made based on knowledge of the current state of the destination station, including users who are on their way to that station with a vehicle, and of all future arrivals of renters to that station (including their exact arrival times). We refer to the system decision process as *look-ahead*, since the system's decision is based on anticipating whether there will be an available parking space in the destination station upon arriving to it. The look-ahead algorithm is presented in Table 2. We use the following notation to describe it:

E	A list of future events in the returning station, including return events of reservations that were already approved and all future rent events. The list is sorted in ascending order of time, where each event is of the form (time, type)
$x.time$	The time of event x
$x.type$	The type of event x
rt	Return time of the user who is attempting to make a reservation.
lt	The latest returning time of a reservation that was previously made in the return station
O	Occupancy at the return station (parked and waiting vehicles)
C	Capacity of the return station

Table 2: Look-ahead algorithm

Input: (E, rt, lt, O, C)

1. While $x.time < rt$
 - If $x.type = rent$ and $O > 0$, set $O = O - 1$.
 - If $x.type = return$, set $O = O + 1$.
 - Set x to the following event in the list
2. Set $O = O + 1$ (for current user)
 - If $O > C$, go to 5.
3. If $lt > rt$
 - While $x.time \leq lt$
 - If $x.type = rent$ and $O > 0$, set $O = O - 1$.
 - If $x.type = return$, set $O = O + 1$.
 - If $O > C$, go to 5.
 - Set x to the following event in the list
4. Return "Reservation Allowed".
5. Return "Reservation Denied".

Interestingly, in some cases under this utopian overbooking policy, users may arrive at their returning station and find that there is no vacant parking space to return the vehicle to. This may occur because in the look-ahead algorithm, it is assumed that all future demand for journeys outgoing from the destination station will reduce the occupancy of the station. But, some renters may decide to abandon the system due to their inability to make a reservation at their destination and therefore the occupancy of the station may be higher than anticipated. Here, the system is not penalized for shortages of parking spaces. Instead, the users are assumed to leave the system at their destination as if they were allowed to park their vehicles near the station. In other words, the station capacity is increased artificially until a parking space becomes available. Therefore, all users who made reservations are guaranteed that they will be able to return their vehicles upon arrival at their destination.

We note that in a real stochastic setting, overbooking is likely to lead to more shortage events than in this utopian policy since the demand forecast is less accurate. Moreover, in reality, when shortage events occur, users are not allowed to leave their vehicles near the stations. Instead, they will have to waste more time in search for a vacant parking space or to wait for a parking space to become available. Therefore, under an actual overbooking policy, the total excess time is likely to be higher than in our utopian overbooking policy.

4. Numerical study

In this section, we evaluate the proposed partial reservation policies with various threshold parameters and demand characteristics, using discrete event simulation of vehicle sharing systems. The simulation is based on the user behavior model presented in Section 3.1. The results are compared to the lower bounds that were devised in Section 2. The numerical study is based on data from two real world bike sharing

systems, Capital Bikeshare and Tel-O-Fun. In Section 4.1 we describe the two bike sharing systems and the trip data that was used to generate the input to our models. In Section 4.2 we present the results of the numerical experiments and discuss their implications.

4.1. Case studies

The Capital Bikeshare system was launched on September 2010. The system operates in Washington D.C., Arlington County and Alexandria, Virginia and Montgomery County, Maryland. The operating Company, Alta Bicycle Share, provides full trip history data, which can be downloaded from the following link: <http://capitalbikeshare.com/trip-history-data>. In this study we use trip data from the second quarter of 2013. In that period there were 232 operative stations with 3860 parking spaces and about 1750 bicycles in the system. The average number of daily trips on weekdays was about 7800.

In Figure 2 we present a map with the stations that were operative in that period. On the map we mark three clusters of stations: Arlington, Alexandria and Crystal City. As can be observed, in these clusters the stations are scattered densely while they are relatively distant from other stations in the system. Indeed, most of the trips that originated or ended in these clusters were within the cluster. In Alexandria, about 90% (resp., 88%) of the journeys that originated (resp., ended) in the cluster ended (resp., originated) in the cluster. In Crystal City, the figures are 77% and 74% and in Arlington, 70% and 76%, respectively. In the following section, we present results for the entire system and for each of the three clusters separately. While generating the data for each of the clusters, we neglected trips from/to other stations in the system. Though the resulting data does not fully reflect the occurrences in these stations, it allows us to analyze small systems in varying sizes which are “close to real”.

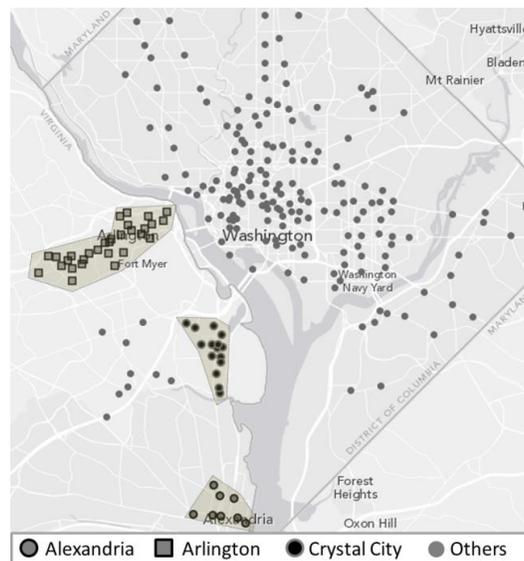


Figure 2: Map of Capital Bikeshare stations (2nd quarter 2013)

The second system that was studied is the Tel-O-Fun bike sharing system in Tel-Aviv. The system was launched on April 2011 and the trip data was collected during a period of two months in the beginning of 2012. At that time, the system consisted of 130 stations scattered in an area of about 50 square kilometers, a total of 2500 parking spaces and about 900 bicycles. During this period, the average number of daily trips (on weekdays) was about 4200.

The input for the simulation for both systems was generated as follows. We assumed that the alternative mode of transportation is walking, which we believe is typically the case for bike sharing systems. The riding times and walking times were estimated using the Google Maps API. The capacities of the stations were retrieved from the systems' websites. The arrival rates of renters during 30 minutes periods throughout the day were estimated by aggregating the weekday trips. Assuming Poisson demand processes, for each system we randomly generated 50 daily demand realizations including renters' arrival times to each station and their destinations. In order to reduce variation, we used the same realizations for all examined policies (Common random numbers). In addition, for each demand realization, we generated the input for the PR-LB and PRP-LB models, namely the set of potential itineraries per realized journey.

Two methods for setting the initial inventory level of vehicles at the stations were used: (1) the actual initial station inventories on a randomly chosen day, after the operators executed repositioning activities; (2) the initial inventory levels prescribed by the method of Raviv and Kolka (2013).

4.2. Results

The discrete event simulation, together with the user behavior model and the preprocessing of the input for the mathematical models, were coded in MathWorks Matlab[™]. The PR-LB and PRP-LB models were solved using IBM ILOG CPLEX Optimization Studio 12.5.1. The codes and data are available upon request from the authors.

We begin by discussing the results of the lower bounds and the utopian overbooking policy. The results and discussion regarding the partial reservation policies are presented at the end of this section. In Table 3 we present results for the Capital Bikeshare and Tel-O-Fun systems. In the first and second columns, the name of the system and the number of stations in the system are given. In the third column we present the method according to which the initial inventory levels were set. In the fourth to seventh columns we present the average total excess time, over 50 realizations, for the NR policy, the CPR policy, the utopian overbooking policy and the PR-LB model. In the last column we present the average total ideal times, over 50 realizations. Recall that the ideal time is the total traveling time if all the journeys could be served ideally by a shared vehicle from the desired origin to the desired destination. The problem instances for the PRP-LB model could not be solved using the available computational resources and thus

this lower bound is not presented here. We revisit this model when analyzing the smaller sub-systems below.

Table 3: Results for the two real-world systems

System	Stations	Initial Inventory	Total Excess Time (hours/day)				Total Travel Time (hours/day)
			NR	CPR	Over-booking	PR-LB	Ideal
Capital Bikeshare	232	Actual Day	346.9	282.4	271.5	114.7	1,347.0
		Raviv & Kolka	183.9	141.1	132.1	58.3	
Tel-O-Fun	130	Actual Day	89.9	76.4	75.8	23.7	919.9
		Raviv & Kolka	59.5	41.2	38.9	15.4	

We observe from Table 3 that the lower bound on the total excess time provided by the PR-LB model is significantly tighter than the trivial lower bound obtained by assuming that all the journeys are materialized by their ideal itineraries, i.e., no excess time, as in Kaspi et al. (2014). For example, in Capital Bikeshare, about 40% of the gap between the CPR policy and the trivial lower bound (zero excess time) is explained by the PR-LB model. That is, at least 40% of the excess time under the CPR policy, cannot be reduced by any passive system regulation.

Using the initial inventories as prescribed by the method of Raviv and Kolka (2013), a significant reduction in excess time is obtained, as can be observed for all policies. Indeed, proper planning of static repositioning results with a major improvement in the service level. Nevertheless, the results for the CPR policy and the PR-LB model suggest that an additional substantial reduction in the total excess time can be achieved by integrating repositioning activities with an efficient passive regulation.

As can be observed in Table 3, the utopian overbooking policy produced only slightly better results as compared to the CPR policy. This is quite surprising, when recalling the assumptions on which the utopian overbooking policy is based on. That is, even with full knowledge of the demand realizations and the use of overbooking, a significant improvement cannot be obtained. This implies that, realistic overbooking policies are not likely to be significantly (or at all) beneficial in terms of reducing the excess time in VSS. This unexpected finding can be explained by the fact that in VSS, a positive side-effect of parking space reservations is the diversion of the demand toward less congested stations. This in turn may have a positive effect on future users of the system who are less likely to face shortages of vehicles and parking spaces. Allowing overbooking reduces this positive side-effect. Given the fact that a good overbooking policy is much more complicated to implement than the CPR policy, and that it also introduces additional uncertainty and thus reduces the trust of the users in the system, we believe that this type of policy should not be practiced in VSSs.

Solving the PRP-LB model that was presented in Section 2.3 is impractical for large real-world systems, due to the large number of binary variables. To obtain the insights provided by the solution of

the PRP-LB model, we generated three small sized systems based on three clusters of stations in the Capital Bikeshare system, namely, Alexandria with 8 stations, Crystal City with 15 stations and Arlington with 30 stations. In Table 4, we present the results for these systems. The table is supplemented with an additional column (the seventh), presenting the lower bound on the expected total excess time produced by the PRP-LB model. We observe from the table, that for the three small sized systems, the value obtained from the PR-LB model explained about 56-66% of the gap from the trivial (zero) lower bound. However, a larger portion of this gap, namely 67%-81% was explained by the PRP-LB value. This result further strengthens our belief that no parking reservation policy is likely to result in a dramatic (if at all) improvement over the CPR policy. We also note that for these systems, the excess time for the utopian overbooking policy is sometimes slightly higher than that of the CPR policy.

Table 4: Results for the three clusters in Capital Bikeshare

System	Stations	Initial Inventory	Total Excess Time (hours/day)					Total Travel Time (hours/day)
			NR	CPR	Over-booking	PRP-LB	PR-LB	Ideal
Arlington	30	Actual Day	2.907	2.262	2.257	1.548	1.256	28.908
		Raviv & Kolka	1.600	1.129	1.137	0.800	0.660	
Crystal City	15	Actual Day	1.314	1.120	1.117	0.902	0.738	10.689
		Raviv & Kolka	0.656	0.564	0.562	0.431	0.375	
Alexandria	8	Actual Day	0.589	0.352	0.347	0.261	0.214	9.119
		Raviv & Kolka	0.225	0.184	0.183	0.124	0.105	

Next, we consider the partial reservation policies that were presented in Section 3.2., and examine whether they can improve the performance obtained by the CPR policy. In Figure 3, we present the simulation results for these policies. The figure contains six graphs, one for each combination of the two studied real-world systems and the three partial policies. In each graph two curves are displayed, representing the percentage of excess time obtained using the two methods for setting the initial inventories. Namely, an actual day, displayed in black, and the method of Raviv and Kolka (2013), displayed in gray.

For each partial policy, we plot the percentage of excess time (relative to the ideal time) under various settings. For the trip based partial policy, we tested various time thresholds. For the time limited partial policy we tested various time limits. For the station based partial policy, we tested various difference thresholds during one hour time intervals. In order to use the same scale in the horizontal axis for the two systems, we present the percentage of stations in which a parking space reservation is required, instead of presenting the difference thresholds.

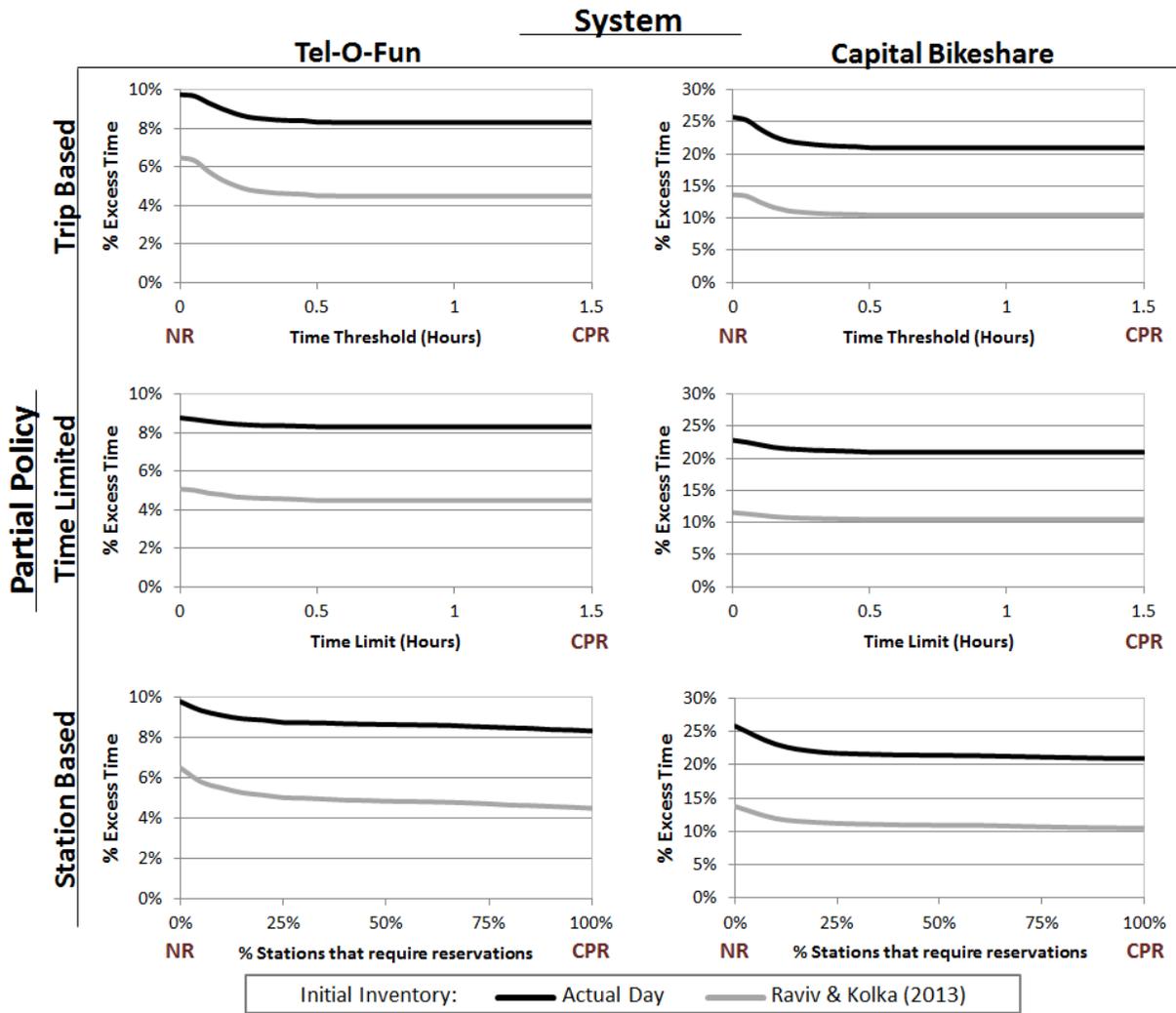


Figure 3: Partial reservation policies - percentage excess time under various settings

Recall that extreme settings of such partial policies result with the complete policies (except for the lower extreme of the time limited partial policy). The figure demonstrates that as the time threshold increases, the same trend appears in all six graphs, i.e. when more reservations are required, the excess time decreases. The best performance is achieved when parking reservations are fully enforced, i.e., under the CPR policy.

These results demonstrate that using a simple rule to define a partial parking reservation policy is not likely to produce better results than the CPR policy. Another outcome of this analysis is that the more parking reservations are enforced, the better the performance of the system is. However, in situations where parking reservations cannot be fully enforced, it would be better to enforce them partially than not at all.

Recall that in the time limited partial reservation policy (Section 3.2.3), all users must make a reservation but the reservation expires after a given time. If the time limit is set to zero, users would only

be able to travel to stations that are not full at the renting time, but a parking space will not be guaranteed in the destination in any case. In Figure 3, the graphs of the time limited policy begin at lower points, compared with the other two partial policies. That is, as compared to the NR policy, it is observable that a significant improvement may be obtained simply by redirecting users to returning stations that are not full at the renting time. In fact, most of the improvement accomplished by the CPR policy may be attributed to the redirection of users to stations with vacant parking spaces.

5. Concluding remarks

The main message of this study is reinforcing the effectiveness of parking reservations in VSS as a method to improve the service provided to its users. It is shown that the simplest possible parking reservation policy (namely, CPR) appears to be the most effective one in terms of reducing the total excess time. Using a lower bound calculated by the PR-LB model, we have demonstrated that, in our case studies, a significant share of the excess time that could be theoretically saved by any passive system regulation, is already saved by the CPR policy. Our extended PRP-LB model shows that other parking reservation policies are not likely to be able save substantially (if at all) more excess time.

We also studied several partial reservation policies and demonstrated that while these policies are slightly inferior to the CPR, they may also be a good alternative to the basic NR policy in cases where CPR cannot be implemented for some reason. Finally, we precluded reservation policies that are based on overbooking as a parking reservation approach that is likely to outperform the CPR policy. This was achieved by showing that even under a utopian scenario in which the system looks ahead into future demand, such policies cannot significantly reduce the excess time obtained by CPR.

The PR-LB based lower bound, introduced in Section 2, can be used to evaluate the effectiveness of various other VSS related policies. This model reflects the fact that each journey may be assigned to one of several itineraries. This adds a lot of flexibility to VSSs and affects its dynamics in a way that should not be ignored by a strategic planner. Though we have focused on reducing the excess time of users, our model can be easily extended to accommodate other objectives of the commuters. That is, each potential itinerary can be assigned with a measure that reflects a combination of several objectives.

While the PR-LB model is formulated in order to evaluate the effectiveness of passive regulations, with some slight modifications it can be used to solve other planning problems in VSS. For example, the initial inventories and station capacities are given as an input to the model. However, their optimal values can be decided by the model. That is, the model can assist with planning of the expansion of stations and the proper initial vehicle inventories. We leave such extensions to future research.

In the numerical experiment, in the Capital Bikeshare instance, 200 potential itineraries per user were generated on average. The resulting models consisted with 5 million continuous variables on average (and

the number of constraints was in the same order of magnitude). To solve larger instances, the average number of potential itineraries per user must be significantly smaller. An interesting direction for future research is to devise a row and column generation algorithm to quickly solve larger instances of this problem. We note that if the model is used for determining the initial inventories or capacities of the stations, reducing the size of the resulting models becomes even more essential. Since these decisions are mutual for all demand realizations, all demand realizations will need to be solved simultaneously in the same model, thus generating even larger models.

Acknowledgments

This research was supported by the Israel Science Foundation (ISF) grant no 1109/11.

References

- Angeloudis, P., Hu, J., Bell, M-G-H, 2012. A strategic repositioning algorithm for bicycle-sharing schemes. In: *Transportation Research Board 91st Annual Meeting Compendium of Papers*.
- Benchimol, M., Benchimol, P., Chappert, B., Taille, A-D-L., Laroche, F., Meunier, F., Robinet, L., 2011. Balancing the stations of a self-service "Bike Hire" system. *RAIRO Operations Research*, 45, 37–61.
- Chemla, D., Meunier, F., Wolfer-Calvo, R., 2013a. Bike sharing systems: solving the static rebalancing problem. *Discrete Optimization*, 10(2), 120-146.
- Chemla, D., Meunier, F., Pradeau, T., Wolfer Calvo, R. and Yahiaoui, H. 2013b. Self-Service Bike Sharing Systems: Simulation, Repositioning, Pricing. Working paper. http://hal.archives-ouvertes.fr/docs/00/82/40/78/PDF/RealTime-BikeSharing_final.pdf.
- Contardo, C., Morency, C., Rousseau, L-M., 2012. Balancing a dynamic public bike sharing system. <https://www.cirrelt.ca/DocumentsTravail/CIRRELT-2012-09.pdf>
- Correia, G., Antunes, A., 2012. Optimization Approach to Depot Location and Trip Selection in One-Way Carsharing Systems. *Transportation Research Part E*, 48, 233–247.
- DeMaio, P., 2009. Bicycle-Sharing: History, Impacts, Models of Provision, and Future, *Journal of Public Transportation*, 12(4), 41-56.
- DeMaio, P., Meddin, R., 2014. The Bike Sharing World Map. <http://bike-sharing.blogspot.com>.
- Erdoğan, G., Laporte, G., Wolfer-Calvo, R., 2013. The One-Commodity Pickup and Delivery Traveling Salesman Problem with Demand Intervals. Working paper
- Forma, I., Raviv, T., Tzur, M., 2014. Optimization based Heuristic for Large Scale Static Repositioning Problems. Working paper, Tel-Aviv University.
- Fricke, C., Gast, N., 2014. Incentives and regulations in bike sharing systems with stations of finite capacity. <http://arxiv.org/pdf/1201.1178v4.pdf>
- George, D-K., Xia, C-H., 2011. Fleet-sizing and service availability for a vehicle rental system via closed queueing networks. *European Journal of Operational Research*, 211(1), 198-217.
- Kaspi, M., Raviv, T., and Tzur, M., [Parking reservation policies in one-way vehicle sharing systems](#). *Transportation Research Part B: Methodological*, 62 (2014), 35-50.
- Kek, A., Cheu, R., Meng, Q., Fung, C., 2009. A Decision Support System for Vehicle Relocation Operations in Carsharing Systems. *Transportation Research Part E*, 45, 149–158.
- Kloimüllner, C., Papazek, P., Hu, B. and Raidl, G.R. (2014). Balancing Bicycle Sharing Systems: An Approach for the Dynamic Case. In: C. Blum and G. Ochoa (Eds.): EvoCOP 2014, LNCS 8600, 73–84.

- Lin, J-R., Yang, T-H., 2011. Strategic design of public bicycle sharing systems with service level constraints. *Transportation Research Part E*, 47, 284–294.
- Lin, J-R., Yang T-H., Chang, Y-C., 2011. A hub location inventory model for bicycle sharing system design: Formulation and solution. *Computers & Industrial Engineering*. Doi:10.1016/j.cie.2011.12.006
- Nair, R., Miller-Hooks, E., 2011. Fleet management for vehicle sharing operations. *Transportation Science*, 45(4), 524–540.
- Pessach, D., Raviv, T., Tzur, M., 2014. Dynamic Repositioning in a Bike Sharing System. Working Paper, Tel-Aviv University.
- Pfrommer, J. Warrington, J., Schildbach, G. and Morari, M. (2013). Dynamic vehicle redistribution and online price incentives in shared mobility systems. Working paper. <http://arxiv.org/pdf/1304.3949v2.pdf>.
- Raviv, T., Kolka, O., 2013. Optimal Inventory Management of a Bike Sharing Station. *IIE Transactions*, 45(10), 1077-1093.
- Raviv, T., Tzur, M., Forma, I., 2013. The static repositioning problem in a bike sharing system: Models and Solution Approaches. *EURO Journal of Transportation and Logistics*, 2(3), 187-229.
- Schuijbroek, J., Hampshire, R., van Hoes, W-J., 2013. Inventory Rebalancing and Vehicle Routing in Bike Sharing Systems. *Tepper School of Business*. Paper 1491. <http://repository.cmu.edu/tepper/1491>
- Shaheen, S., Cohen, A-P., 2007. Growth in Worldwide Carsharing: An International Comparison. *Transportation Research Record*, 1992, 81-89.
- Shaheen, S., Cohen, A., 2012. Innovative Mobility Carsharing Outlook, Fall 2012. Transportation Sustainability Research Center, University of California, Berkeley.
- Shaheen, S., Guzman, S., 2011. Worldwide Bikesharing. *Access*, 39, 22-27.
- Shaheen, S., Guzman, S., Zhang, H., 2010. Bicycle Sharing in Europe, the Americas, and Asia: Past, Present, and Future. *Transportation Research Record*, 2143, 159-167.
- Shu, J., Chou, M., Liu, Q., Teo, C-P., Wang, I-L., 2013. Models for Effective Deployment and Redistribution of Bicycles within Public Bicycle-Sharing Systems. *Operations Research*, 61(6), 1346-1359.
- Vogel, P., Saavedra, B. A. N., & Mattfeld, D. C. (2014). A hybrid metaheuristic to solve the resource allocation problem in bike sharing systems. In *Hybrid Metaheuristics* (pp. 16-29). Springer International Publishing.
- Waserhole, A., Jost, V., Brauner, N., 2013. Pricing techniques for self regulation in Vehicle Sharing Systems. *Electronic Notes in Discrete Mathematics*, 41, 149-156.